# A preliminary, calibrated scheme for estimating rock mass properties for non-linear, discontinuum models

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ABSTRACT: Calibration of 3D, discontinuum, strain softening, dilatant (SSD) models of mines often yields different rock mass scale (representative elementary volume, or REV) properties to empirical methods. In this paper, some steps towards a calibrated empirical scheme for estimating material properties for some types of 3d, discontinuum non-linear models targeting larger than (REV) scale phenomena are described. The scheme uses typical pre-mining rock mass classification date and strength tests to estimate the peak and residual yield, softening and dilatancy parameters. The scheme is a purely empirical device, derived from UCS and GSI field data and calibrated model results, and is a work in progress. The underlying nature of the Hoek Brown-GSI scheme (Hoek, E., and E. T. Brown., 1997; Hoek  et al., 2002), is validated by the work.

## 1 INTRODUCTION

An essential task of modelling of rock masses is to establish the length scale below which each particular rock mass can be treated as a continuum. On this scale the medium is said to be homogeneous (Witherspoon et al, 1981). Witherspoon et al (1981) illustrated this with a diagram for permeability similar to the one shown in Figure 1.

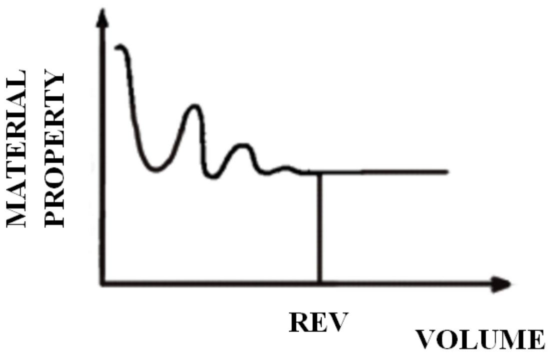



Figure 1. Change in value of measured properties with size of sample. REV = the Representative Volume Element

The REV of rocks is ordinarily large enough to preclude laboratory measurements of REV scale properties, so calibration or an empirical estimate are necessary. Calibration is usually preferred.

Quantitative calibration is the process of adjusting model inputs to achieve a like-for-like match between measured data and model results.  The quality of calibration qualifies the model for use and is measured by the resolution, precision and coverage of the models match to real measurements across space, time and length scales.

When calibration is not possible, REV scale properties must be empirically estimated. The most commonly used empirical scheme for estimating ‘rock mass scale’, or REV strengths for non-linear geotechnical models is the Hoek-Brown Geological Strength Index (HB-GSI) after Hoek, E., and E. T. Brown (1997). That scheme uses the laboratory scale unconfined compressive strength (UCS), a qualitative classification of the nature of the discontinuity network (GSI) and a parameter obtained from triaxial testing of laboratory specimens ($m_i$) to establish the parameters of the yield potential function.

The HB-GSI approach has been widely applied to the problem of estimating REV properties, but only explicitly forecasts parameters of the peak strength yield criterion. It can be extended to estimate intermediate and residual rock mass yield parameters (eg, Martin et al., 1999) by making some logical assumptions, but there are no extensive data for that in the literature.

The calibrated scheme we propose here is for strain softening, dilatant, discontinuum models targeting realistic deformation. The scheme produces most of the inputs required for models of this type. It is not a replacement for calibration and where field data is available to calibrate a model, results derived that way should take precedence.

The current state of the project is a ‘work in progress’ and has highlighted some challenges, especially the limits of GSI, UCS and $m_i$ for differentiating all necessary rock mass properties. The subjective nature of GSI is a particular concern that must be addressed in future iterations of the scheme and users of the scheme must consider the reliability of this classification at their site.

## 2 CONTRAINDICATIONS

This scheme is only for the first stages of a geotechnical simulation project. Continuous updates to model resolution and, ultimately, calibration as data comes to hand is essential.

The scheme only applies to properly formulated, 3d, discontinuum, strain softening models of high resolution and is for estimating REV scale, not smaller scale properties. For rock properties at smaller scales, an alternative approach is necessary. 'Down-scaling' of calibrated REV scale properties is one possibility. See for example the procedure for down-scaling of calibrated REV scale properties described in Beck et al (2009).

Users should consider the particular constitutive model used in the case studies and determine if their formulation is a sufficient match. Models using similar constitutive approaches may be able to use the outputs of this scheme.

We recommend against using the scheme to estimate material properties for interpreting elastic models.

The highest confidence in the scheme is for moderate to strong rock masses, as that is where the bulk of data is from. For weak rock masses, the softening parameters are an extrapolation from stronger rocks. When using the scheme for weak rocks, the user should also consider that various estimates of the material properties are possible using the scheme and that the variance between these estimates is large relative to the magnitude of the values.

## 3 THE WORKFLOW

The work flow was as follows:

- Models were calibrated to sufficiently replicate field measurements.
- The calibrated rock mass properties were then compared to the pre-mining measured UCS and GSI to derive best fit functions: model parameters = f (UCS, GSI)
- The measure of sufficiency of the functions used in the optimisation process is the closeness of the fit between the 'forecast' values (those derived by the functions) and the real data (the calibrated values).

## 4 CONSIDERATIONS

### 4.1 *Candidates mines for inclusion in the scheme*

Mine models were selected for inclusion on the following basis:

- The models were designed to capture behaviour at scales larger than the REV. To capture behaviour at scales smaller than the REV, such as bulking of a tunnel wall, requires a higher resolution discontinuum approach.
- A high density of deformation, damage or seismicity measurements across connected lithological domains, spanning a range of strains from minor to very significant were available.
- A high quality, 3D interpretation of relevant geological structure.
- Reliable rock mass classification data for all the relevant domains
- A detailed history of mining
- A sufficient number of rock stress measurements to understand the variability

### 4.2 *Field data*

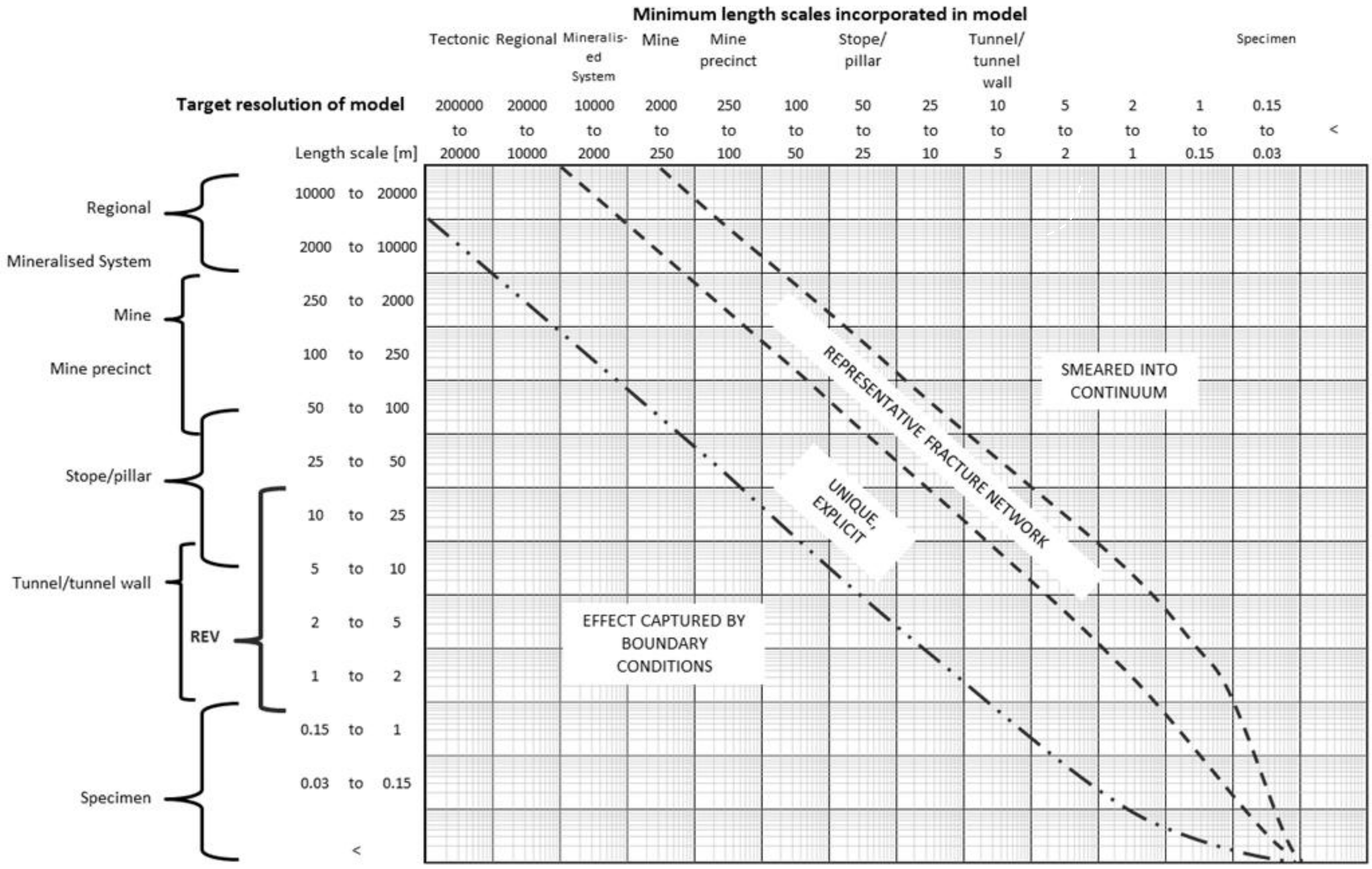
For this study, the pre-mining data collected prior to the study by mines themselves were taken at face value. This means that the sampling and interpretation biases are retained in the data. As our effort is to develop a scheme for converting a mines 'real' pre-mining data into material properties, this is not necessarily inappropriate. Obviously incorrect data should be removed, but random variability and indeed erroneous, but plausible entries, introduced by the normal population of diligent data collectors are a fact of the problem.

At this time, the pre-mining data available for calibration are GSI, $m_i$ and UCS measurements. Each has its own problems, including:

- Logically, the purpose of $m_i$ in the HB-GSI scheme is valid and it should improve reliability if it were estimated correctly. However, achieving a representative value by laboratory testing is extremely problematic. Few mines collect it.
- GSI data are widely available, but these are sometimes unfortunately also estimated by 'reckoning' rather than by a consistent process.

### 4.3 *Model types*

The framework used in all the case studies was Levkovitch-Reusch 2 (Levkovitch et al, 2010 and Reusch et al, 2010). This framework can be replicated using a number of different discontinuum modelling packages and computational approaches but has a specific strain softening dilatant constitutive model for the continuum parts.




| Method of incorporating structure | Explanation |
|---|---|
| Smeared into continuum | Incorporated into the continuum constitutive material model |
| Representative fracture network (eg, DFN) | A distribution of explicit discontinuities, in 3d, matching the distribution measured in the real rock mass can be used if there is no explicit structural model for this length scale. A unique, explicit interpretation of discontinuities is preferred |
| Unique, Explicit | Faults built in 3d to match the structural geologists interpretation. The 3d geometry of these faults is not substantially simplified |
| Effect captured by boundary conditions | Captured only by the effects of these structures on the displacements used to load the boundaries of the model |

Figure 2. A guide to the method of including discontinuities of different length scales in a geotechnical numerical model, based on the scale of the phenomena that are being targeted.

### 4.4 *Scale and structure*

The REV is a rock mass property and must be estimated for all the domains of a particular problem. For problems targeting deformation phenomena at larger than REV scales, the effects of structures smaller than the REV will be approximated by the REV continuum, while larger structures need to be represented, or accounted for explicitly in some way.

Figure 2 provides an initial guide for representing structures in discontinuum models. Put simply, if the purpose of a model is to estimate behaviour at a certain length scale, how must discontinuities of each length scale range be represented? A line drawn from left to right at the target scale shows the method for incorporating structures of each length scale that line crosses, in that model. The four approaches, usually used in combinations within a single model are explained in the Figure.

This guide is ‘flexible’ and not intended to be prescriptive. When using the guide, the specific circumstances of a modelling task must be accounted for. One should note that the graph extends on both axes to sub REV scale, whereas the empirical scheme does not estimate sub REV scale properties. For sub REV scale modelling, joints and other small discontinuities must be modelled explicitly to generate realistic damage and deformation.

An example model geometry for an open pit, underground interaction problem is shown in Figure 3. The modelling objective was to forecast subsidence around a pit. The required model resolution is implied by the purpose for which the model was built, the situation and the governing physics of cave propagation and stability of the pit walls. Assuming inter-ramp scale instability must be simulated:

- the model must incorporate structures explicitly down to a persistence *less* than inter-ramp scale.
- Structures from an inter-ramp scale upwards were built as per the expert-interpreted structural model.
- Structures with persistence from inter-ramp to just less than inter-ramp scale were built using a DFN, as a complete structural interpretation of structures at that length scale was not available. The DFN is not designed to generate an explicit high resolution forecast. Rather, the DFN is used to augment the global similitude of the model, by assisting to evolve discontinuous phenomena. As the DFN generates small scale phenomena for global effect, few, or fewer realisations of it should be necessary.
- The effects of structures with persistence smaller than inter-ramp scale were smeared into the continuum constitutive model

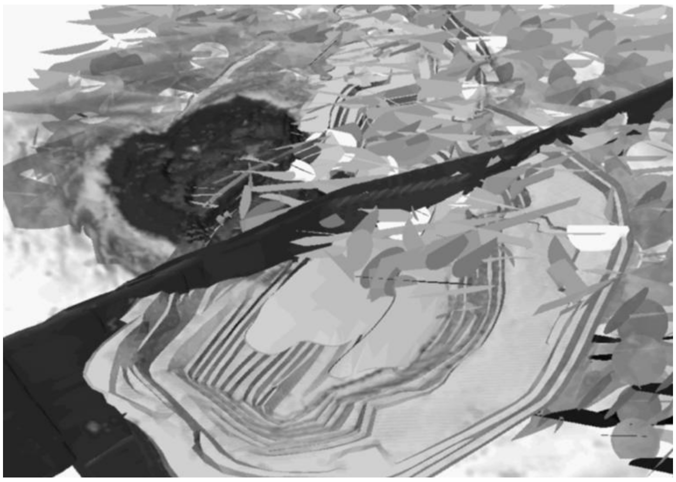


Figure 3. An example of the resolution of discrete, explicit discontinuities that can be simulated using and LR2, FE model, at a mine used as data for the scheme. The image is after Levkovitch et al 2013.

Figure 3 shows a cut-away view of the DFN and larger scale, explicit structures built into the model to meet these specifications.

### 4.5 *Sufficiency of calibration*

The case studies involved calibration of sophisticated models, to quantitatively replicate the extent and magnitude of seismicity, deformation rock mass damage, tunnel closure, movement or surveyed instability. None of the calibrations were based on indirect, qualitative interpretations of isolated failures, such as simple correlations of 'high' relative movements ('red' areas) or any subjective measure of 'high stress' compared to equally subjective damage.

To be considered calibrated, a rock mass domain in the model must have exhibited deformation, damage or seismicity that matched similar field observations over the calibration time window and a range of circumstances. The calibrations must also match non-events – the null hypothesis must have been tested and sufficiently satisfied by the model forecasts. In most cases, a match to all of these measures (seismicity, damage, displacement and instability) was achieved. The calibration measures are discussed by Reusch et al. (2010) and Levkovitch et al. (2013).

## 5 ROCK MASS PROPERTIES AS FUNCTIONS OF PRE-MINING CLASSIFICATION DATA

### 5.1 *Peak properties*

The Hoek -Brown yield criterion (Hoek and Brown, 1998) is given by:

$$\sigma_1 = \sigma_3 + \sigma_{ci} \left\{ m_b \frac{\sigma_3}{\sigma_{ci}} + s \right\}^a \quad (1)$$

Where $\sigma_1$ and $\sigma_3$ are the major and minor principal stress, $\sigma_{ci}$, $m_b$, s and a are material constants that can be related to GSI and the rock mass damage D (Hoek et al., 2002):

$$s = e^{\left(\frac{GSI-100}{9-3D}\right)} \; ; \; m_b = m_i \, e^{\left(\frac{GSI-100}{28-14D}\right)} \quad (2,3)$$

Our main effort is to estimate $m_b$ and s using data available prior to mining. In the current scheme, the disturbance parameter D was not estimated by any of the mines in a formal manner, so was unavailable for inclusion, and $m_i$ is practically difficult to obtain as discussed above. Thus, our pre-mining data were limited to GSI and UCS.

We note that adding disturbance parameter D and $m_i$ or similar values to the scheme we describe here would be valuable and is the subject of future work, but was not possible at this time.

A function considering both GSI and UCS gave the best correlation between 'predicted' peak $m_b$ and s and calibrated $m_b$ and s values. The best fit functions of UCS and GSI for $m_b$ and s were:

$$m_b = \frac{GSI}{100} e^{0.01UCS} \qquad R^2 = 0.88 \quad (4)$$

$$s = \frac{GSI}{350000} \, e^{0.027UCS} \qquad R^2 = 0.88 \quad (5)$$

'Predicted' in this sense means the $m_b$ and s values that would have been derived using only the correlative functions and those which were derived by actual calibration. A limitation at some mines is that only one of GSI or UCS data is available, so cruder, but still useable correlations between GSI or UCS and peak strength $m_b$ and s were also derived. The graphs are shown in the Reusch et al (2013). The correlations for use when such data, with their correlation coefficients are:

$m_b = 0.573\ e^{\ 0.0106UCS}$ $R^2$=0.82 (6)

$m_b = 0.177\ e^{\ 0.0383GSI}$ $R^2$=0.82 (7)

$s = 0.0003\ e^{\ 0.0202UCS}$ $R^2$=0.70 (8)

$s = 0.0005\ e^{\ 0.0783GSI}$ $R^2$=0.82 (9)

The Peak Youngs Modulus (E) and *d*, the dilatancy parameter of the plastic strain potential were best described using the following:

$E = 209\ UCS$ $R^2$=0.88 (10)

$d = 0.0024\ UCS$ $R^2$=0.74 (11)

$d = 0.0127\ e^{\ 0.045GSI}$ $R^2$=0.78 (12)

It is important to note that *d*, is a parameter of the plastic strain potential and not the disturbance D used in the Hoek-Brown Scheme. The plastic strain potential $\boldsymbol{D}_p$ is given by the relation:

$$\boldsymbol{D}_p = \dot{\lambda}\frac{\partial G}{\partial \boldsymbol{\sigma}} \qquad (13)$$

Where $\lambda$ the accumulated equivalent plastic strain and $G$ the flow potential:

$$G = \left[\frac{q}{\sigma_{ci}}\right]^{\frac{1}{a}} + m\left[\frac{1}{3}\frac{q}{\sigma_{ci}}R(\theta,e) - d\frac{p}{\sigma_{ci}}\right]. \qquad (14)$$

Where, $d$ is the dilation parameter of the bulk, $p$ is the hydrostatic pressure and $q$ is the Mises equivalent stress. The variable $\theta$, defined via $\cos 3\theta = (r/q)^3 \cos 3\theta = \left({}^{r}/_{q}\right)^3$ is the deviatoric polar angle and the material constant $e$ is the deviatoric eccentricity that describes the "out-of-roundedness" of the deviatoric trace of the function $R(\theta, e)$ in terms of the ratio between the Mises stress along the extension meridian $(\theta = 0)$ and the compression meridian $(\theta = \pi/3)$. To best fit the Hoek Brown yield criterion to laboratory measurements, *e* = 0.6.

### 5.2 *Residual*

For residual REV properties, combining GSI and UCS did not improve the efficacy of the predictive functions, so functions of GSI or UCS are currently proposed. In future, as information is obtained to populate the schemes database these relations can be refined.

For now:

$m_b = 0.012\ UCS$ $R^2 = 0.82$ (15)

$s = 4\text{x}10^{-8}\ UCS^{\ 2.25}$ $R^2 = 0.88$ (16)

d = 0.0022 GSI $R^2 = 0.34$ (17)

### 5.3 *Softening*

The softening rules in LR2 models are a function of plastic strain, and describe the transition of properties from pre-peak, through yield to residual. Any number of points can be used to define the softening function, but the levels of equivalent plastic strain at which degradation begins (pe2) and at which the residual strength is attained (pe3) are critically important. Example strain softening curves are shown in Figure 3. The correlations are:

pe2 = 0.040 $e^{\ -0.006\ UCS}$ $R^2$=0.80 (18)

pe3 = 0.070 $e^{\ -0.006\ UCS}$ $R^2$=0.80 (19)

We note these functions are derived from only a few data points.

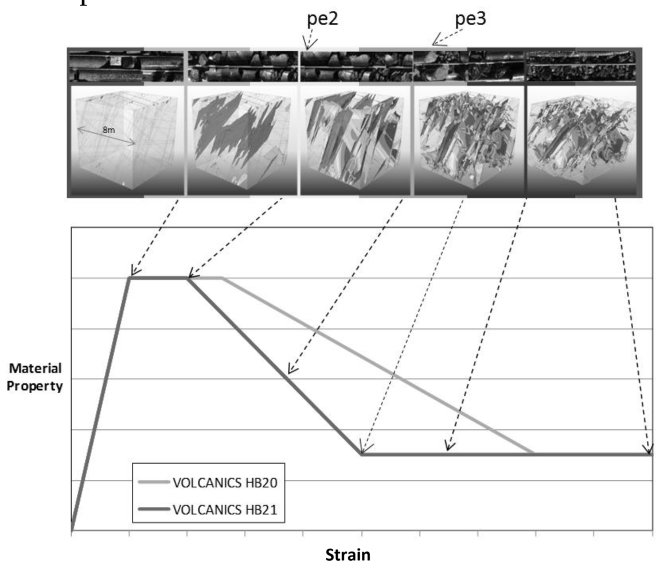



Figure 4. Example softening curves

## 6 NEXT STEPS

### 6.1 *Softening*

At present, all of the case studies used a piece-wise linear approximation of a softening curve to describe the relation between softening and strain. This permitted a simple correlation between pre-mining classification data and points on the softening curve. At present, the pre-mining data are UCS and GSI, but it would likely be more appropriate to incorporate a measure similar to $m_i$ and possible others, alongside these.

We propose that a measurement program during early mining may be used to derive better data. In one of the case studies, the pillar between adjacent tunnels was diamond drilled to observe the levels of damage, and a reliable calibration of the softening parameters was possible. In other studies, convergence measurements in closely spaced tunnels were used.

### 6.2 *Faults*

In all the case studies the fault properties were calibrated along similar lines to the rock mass types. First the faults were classified into groups, and then adjusted in a group basis until a close match to the observed deformation and seismicity was achieved.

The difference for faults was that the classifications are qualitative, often simply ‘weak’ or ‘strong’, so do not lend easily to empirical use in a scheme such as this. Over time, a similar scheme for faults must be developed, but first mines must start to classify faults using quantitative measures.

### 6.3 *Replacing GSI*

GSI values should not be subjective, but in practise, are often estimated without recourse to a reproducible process. We propose that alternative measures might be more reliable. A potential candidate is $P_{21}$, the length of fracture traces per unit area of sampling surface obtained by mapping tunnel or pit walls. $P_{32}$, the area of fractures per unit volume of rock mass, reconstructed by best-fitting 3 wall REV scale mapping of excavations across a domain may be more useful. We have very limited $P_{32}$ data estimated at an appropriate scale, some of it of questionable provenance, but it does produce a suggestive relation for the LR2 dilation parameter, *d*.

### 6.4 *Other refinements*

Some of the correlations between the pre-mining data, and the model inputs are probably confounding. In future iterations of this scheme, a goal will be to eliminate biases, by using only data that is generated by repeatable, standardized procedures. For now the current scheme certainly captures similar systemic biases which need to be resolved over time.

## 7 CONCLUSIONS

REV properties are a considerable source of uncertainty for projects with geotechnical risks. Calibration is the best process for forecasting the behaviour of excavations and rock, but at the beginning of a project, empirical REV properties estimates are necessary. The REV properties scheme we propose here for SSD models, comprises an approach to selecting how discontinuities should be represented in a model, and some empirical relationships for REV properties, as functions of common pre-mining data..

The scheme is useful for estimating REV properties of SSD models, but subject to a number of important limitations. It must not be applied for interpretation of elastic models, 2D models or other models not like those used in the schemes case study data. There are also disconcerting incongruities in the scheme, especially related to correlations between GSI and rock mass parameters. It is highly likely that there are confounding correlations amongst the functions used to describe the REV properties. For now we are limited by the data that mines collect and a next stage will be to target more reliable rock mass and discontinuity classification.

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